CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2014 series

4037 ADDITIONAL MATHEMATICS

4037/13 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2014 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

CAMBRIDGE
International Examinations

1	$a = 3$ $b = 2$ $c = 4$	B1 B1 B1	
2	$x^2 = 16$ or $y^2 - 4y + 3 = 0$	M1	for correct elimination of one variable and attempt to form a quadratic equation in x or y.
	$x = \pm 4$ $y = 1, 3$	A1 A1	
	Points (–4, 1) and (4, 3)		
	Line $AB = \sqrt{8^2 + 2^2}$	M1	for use of Pythagoras theorem
	$= \sqrt{68}$ or $2\sqrt{17}$	A1	allow either form
3 (i)	$\mathrm{n}(A) = 2$ $\mathrm{n}(B) = 3$ $\mathrm{n}(C) = 0$	B1 B1 B1	**B0** for n(2), $\{2\}, \{0\}, \varnothing$, $\{\}$ etc.
(ii)	$A \cup B = \{-1, -2, -3, 3\}$	B1	
(iii)	$A \cap B = \{-2\}$	B1	
(iv)	ξ, 'the universal set', R, 'real numbers', $\{x : x \in \quad\}$	B1	
4 (a)	$\tan x = -\frac{5}{3}$	M1	Correct statement or $\tan x = -1.67$
	$x = 121.0°,\ 301.0°$	A1 A1ft	**A1** for either correct solution **ft** from *their* first solution
(b)	$\sin\left(3y + \frac{\pi}{4}\right) = \frac{1}{2}$	M1	for dealing correctly with cosec and attempt to solve subsequent equation
	$3y + \frac{\pi}{4} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$	A1	for $\frac{\pi}{6}, \frac{5\pi}{6}$, or $\frac{13\pi}{6}$, or $\frac{17\pi}{6}$
	$3y = -\frac{\pi}{12}, \frac{7\pi}{12}, \frac{23\pi}{12}, \frac{31\pi}{12}$	DM1	for correct order of operations
	$y = \frac{7\pi}{36}, \frac{23\pi}{36}, \frac{31\pi}{36}$ (0.611, 2.01 and 2.71)	A1, A1	**A1** for one correct solution **A1** for both the other correct solutions and no others in range.

5 (a) (i)	$\begin{pmatrix}12 & 2 & 1\\ 9 & 3 & 0\\ 8 & 5 & 1\\ 11 & 2 & 0\end{pmatrix}\begin{pmatrix}0.5\\ 0.4\\ 0.45\end{pmatrix} = \begin{pmatrix}7.25\\ 5.70\\ 6.45\\ 6.30\end{pmatrix}$	**M1**	for correct compatible matrices in the correct order. Allow 1 error in each matrix. Allow if done in cents
	or $\begin{pmatrix}0.5 & 0.4 & 0.45\end{pmatrix}\begin{pmatrix}12 & 9 & 8 & 11\\ 2 & 3 & 5 & 2\\ 1 & 0 & 1 & 0\end{pmatrix}$	**DM1**	for a correct method for multiplying their matrices to obtain an appropriate 4 by 1 or 1 by 4 matrix.
	$= \begin{pmatrix}7.25 & 5.70 & 6.45 & 6.30\end{pmatrix}$	**A2,1,0**	**A2** all correct or **A1** 3 correct elements.
(ii)	25.70	**B1**	Allow 25.7
(b)	$\mathbf{Y} = \mathbf{X}^{-1}$ or $\mathbf{Y} = \mathbf{X}^{-1}\mathbf{I}$	**M1**	for matrix algebra
	$\mathbf{Y} = \frac{1}{22}\begin{pmatrix}1 & -4\\ 5 & 2\end{pmatrix}$ or $\begin{pmatrix}\frac{1}{22} & -\frac{4}{22}\\ \frac{5}{22} & \frac{2}{22}\end{pmatrix}$	**A1**	for $\frac{1}{22}\begin{pmatrix} & \\ & \end{pmatrix}$
		A1	for $k\begin{pmatrix}1 & -4\\ 5 & 2\end{pmatrix}$
	Alternative method: $\begin{pmatrix}2 & 4\\ -5 & 1\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $2a + 4c = 1,\ 2b + 4d = 0$ $-5a + c = 0,\ -5b + d = 1$	**M1**	for a complete method using simultaneous equations
		A1	$a = \frac{1}{22}$ and $c = \frac{5}{22}$ or $b = -\frac{4}{22}$ and $d = \frac{2}{22}$
	leading to $= \frac{1}{22}\begin{pmatrix}1 & -4\\ 5 & 2\end{pmatrix}$ oe	**A1**	for correct matrix

6 (i)	$\cos 0.9 = \frac{6}{OC}$ or $\frac{OC}{\sin 0.9} = \frac{12}{\sin(\pi - 1.8)}$ $OC = \frac{6}{\cos 0.9} = 9.652...$	**M1**	for correct use of cosine, sine rule, cosine rule or any other valid method
	or $OC = \frac{12\sin 0.9}{\sin(\pi - 1.8)} = 9.652...$	**A1**	for manipulating correctly to $OC = 9.652(35\ldots)$ Must have 4th figure (or more) for rounding
(ii)	Perimeter = $(0.9 \times 12) + 9.652 + (12 - 9.652)$	**B1** **M1**	for arc length for attempt to add the correct lengths
	$= 22.8$	**A1**	
(iii)	Area = $\left(\frac{1}{2} \times 12^2 \times 0.9\right) - \left(\frac{1}{2} \times 9.652^2 \sin(\pi - 1.8)\right)$	**B1**	for area of sector, allow unsimplified
		B1	for area of isosceles triangle $\frac{1}{2}(9.65(2...))^2 \sin(\pi - 1.8)$ or $\frac{1}{2}(12 \times 6\tan 0.9)$ or $\frac{1}{2}(12 \times 9.65(2...) \times \sin 0.9)$, allow unsimplified.
	$64.8 - 45.36$ $= 19.4$ to 19.5	**B1**	for answer in range 19.4 to 19.5
	Alternative Method:		
	$\frac{1}{2}(12 - 9.652) \times 9.652 \times \sin 1.8$	**B1**	for area of triangle *ACB*, unsimplified
	$\frac{1}{2}12^2(0.9 - \sin 0.9)$	**B1**	for area of segment, unsimplified
	$11.04 + 8.40$ Area = 19.4 to 19.5	**B1**	answer in range 19.4 to 19.5
7	$1 + 2\log_5 x = \log_5(18x - 9)$	**B1, B1**	**B1** for dealing with '1', **B1** for dealing with '2'
	$\log_5 5 + \log_5 x^2 = \log_5(18x - 9)$	**M1**	for a correct use of addition or subtraction of logarithms
	$5x^2 = 18x - 9$ $(5x - 3)(x - 3) = 0$	**DM1**	for elimination of logarithms to form a 3 term quadratic and for solution of quadratic
	$x = \frac{3}{5},\ 3$	**A1**	for both *x* values

8 (i)	$f'(x) = \left(x \times \frac{3x^2}{x^3}\right) + (\ln x^3)$	**M1** **B1**	for differentiation of a product for differentiation of $\ln x^3$
	$= 3 + 3\ln x, \ = 3(1 + \ln x)$	**A1**	for simplification to gain <u>given answer</u>
	or $f(x) = 3x\ln x$	**B1**	for use of $\ln x^3 = 3\ln x$
	$f'(x) = \left(3x \times \frac{1}{x}\right) + 3\ln x$,	**M1**	for differentiation of a product
	$= 3(1 + \ln x)$	**A1**	for simplification to gain <u>given answer</u>
(ii)	$\int 3(1 + \ln x)\mathrm{d}x = x\ln x^3$ or $3x\ln x$	**M1**	for realising that differentiation is the reverse of integration and using **(i)**
	$\int 1 + \ln x\mathrm{d}x = \frac{1}{3}x\ln x^3$ or $x\ln x$	**A1**	
(iii)	$x\ln x - \int 1\mathrm{d}x$ or $\left[\frac{1}{3}x\ln x^3\right] - \int 1\mathrm{d}x$	**DM1**	for using answer to **(ii)** and subtracting $\int 1\mathrm{d}x$ dependent on M mark in **(ii)**
	$[x\ln x - x]_1^2$ or $\left[\frac{1}{3}x\ln x^3 - x\right]_1^2$	**DM1**	for correct application of limits
	$= 2\ln 2 - 2 + 1$ $= -1 + \ln 4$	**A1**	from correct working
9 (a)	$5^p = 625$, so $p = 4$	**B1**	
	${}^4C_1 5^{p-1}(-q) = -1500$ $4 \times 125(-q) = -1500$	**M1**	*their p* substituted in ${}^pC_1 5^{p-1}(-q)$ or in ${}^pC_1 5^{p-1}(-qx)$ unsimplified
	$q = 3$	**A1**	
	${}^4C_2 5^{p-2} q^2 = r$	**M1**	*their p* and *q* substituted in ${}^pC_2 5^{p-2}(-q)^2$ or ${}^pC_2 5^{p-2}(-qx)^2$ unsimplified
	$r = 1350$	**A1**	
(b)	${}^{12}C_3 (2x)^9 \left(\frac{1}{4x^3}\right)^3$	**M1**	for identifying correct term
		DM1	for attempt to evaluate correct expression
	Term is 1760	**A1**	must be evaluated

10 (a)	$\frac{5^x}{5^{2(3y-2)}}=1$ or $\frac{3^x}{3^{3(y-1)}}=3^4$ oe	**M1**	for obtaining one correct equation in powers of 5, 3, 25, 27 or 81
	$x=6y-4$	**A1**	for $x=6y-4$ oe linear equation
	$x=3y+1$	**A1**	for $x=3y+1$ oe linear equation
		M1	for attempt to solve linear simultaneous equations which have been obtained correctly
	Leads to $x=6,\ y=\frac{5}{3}$	**A1**	for both.
(b)	Using the cosine rule: $\left(1+2\sqrt{3}\right)^2=\left(2+\sqrt{3}\right)^2+2^2-4\left(2+\sqrt{3}\right)\cos A$	**M1**	for correct substitution in cosine rule, may use in form of $\cos A=\ldots$
	$\cos A=\frac{\left(13+4\sqrt{3}\right)-\left(7+4\sqrt{3}\right)-4}{-4\left(2+\sqrt{3}\right)}$ oe	**DM1**	for attempt to make cos*A* subject and simplify
	$\cos A=\frac{-1}{2\left(2+\sqrt{3}\right)}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$	**DM1**	for rationalisation.
	$\cos A=-1+\frac{\sqrt{3}}{2}$	**A1**	

11 (i)	$\frac{dy}{dx} = (x+5)2(x-1) + (x-1)^2$	**M1** **A1**	for differentiation of a product, allow unsimplified correct
	$\frac{dy}{dx} = (x-1)(3x+9)$		
	When $\frac{dy}{dx} = 0$	**DM1**	for equating to zero and solution of quadratic
	$x = 1$	**A1**	
	$x = -3$	**A1**	
	Alternative method:		
	$y = x^3 + 3x^2 - 9x + 5$	**M1**	for expansion of brackets and differentiation of each term of a 4 term cubic
	$\frac{dy}{dx} = 3x^2 + 6x - 9$	**A1**	
	When $\frac{dy}{dx} = 0$	**DM1**	for equating to zero and solution of 3 term quadratic
	$x = 1$	**A1**	from correct quadratic equation
	$x = -3$	**A1**	from correct quadratic equation
(ii)	$\int x^3 + 3x^2 - 9x + 5 dx$	**M1**	for correct attempt to obtain and integrate a 4 term cubic
	$= \frac{x^4}{4} + x^3 - \frac{9x^2}{2} + 5x \ (+c)$	**A2,1,0**	**A2** for 4 correct terms or **A1** for 3 correct terms
(iii)	$\left[\frac{x^4}{4} + x^3 - \frac{9x^2}{2} + 5x\right]_{-5}^{1}$	**M1**	for correct substitution of limits 1 and –5 for *their* **(ii)**
	$= \left(\frac{1}{4} + 1 - \frac{9}{2} + 5\right) - \left(\frac{625}{4} - 125 - \frac{225}{2} - 25\right)$		
	$= 108$	**A1**	
(iv)	When $x = -3, y = 32$	**M1**	for realising that the *y*-coordinate of the maximum point is needed.
	$k > 32$	**A1**	